Synopsis of the Ph.D. thesis

1. **Name** **Mushtaq Ahmad Shah**
2. **Registration Number:- VGU/2014/RES/SCH/Math/1001**
3. **Title** **"A Study of Barnett's Conjecture on Hamiltonian graphs"**
4. **Subject: - Mathematics**
5. **Objectives and scope of the Research Investigation**

Everything in our world is linked; cities are linked by street, rail and flight networks. Pages on the internet are linked by hyperlinks. The different components of an electric circuit or computer chip are connected and the paths of disease outbreaks form a network. Scientists, engineers and many others want to analyses, understand and optimize these networks and this can be done using graph theory. Airlines want to connect countless cities in the most efficient way, moving the most passengers with the fewest possible trips at the same time, air traffic controllers need to make sure hundreds of planes are at the right place at the right time and don't crash: an enormous task that would be almost impossible without graph theory.

The aim of this research is to prove about fifty year old conjecture known as Barnett's conjecture which states that "every cubic planar three connected bipartite graph is Hamiltonian" This is an open problem in mathematics since 1969. In addition to this we give necessary condition for cubic planer three connected graph to be non Hamiltonian. We also extend Euler's result of number of regions in planar graphs to all simple non planar graphs in what so ever way we would like to draw the simple non planar graph. We will show that result is true for complete graphs are not. We shall also try to find out minimum number of regions in any complete graph. Further we shall try to prove that every hypo Hamiltonian graph is Hamiltonian if we make degree of removable vertex V exactly equal to n-1 and other related results on cubic planar three connected bipartite graphs as well as Hamiltonian and hypo Hamiltonian graphs.

6. Methodology

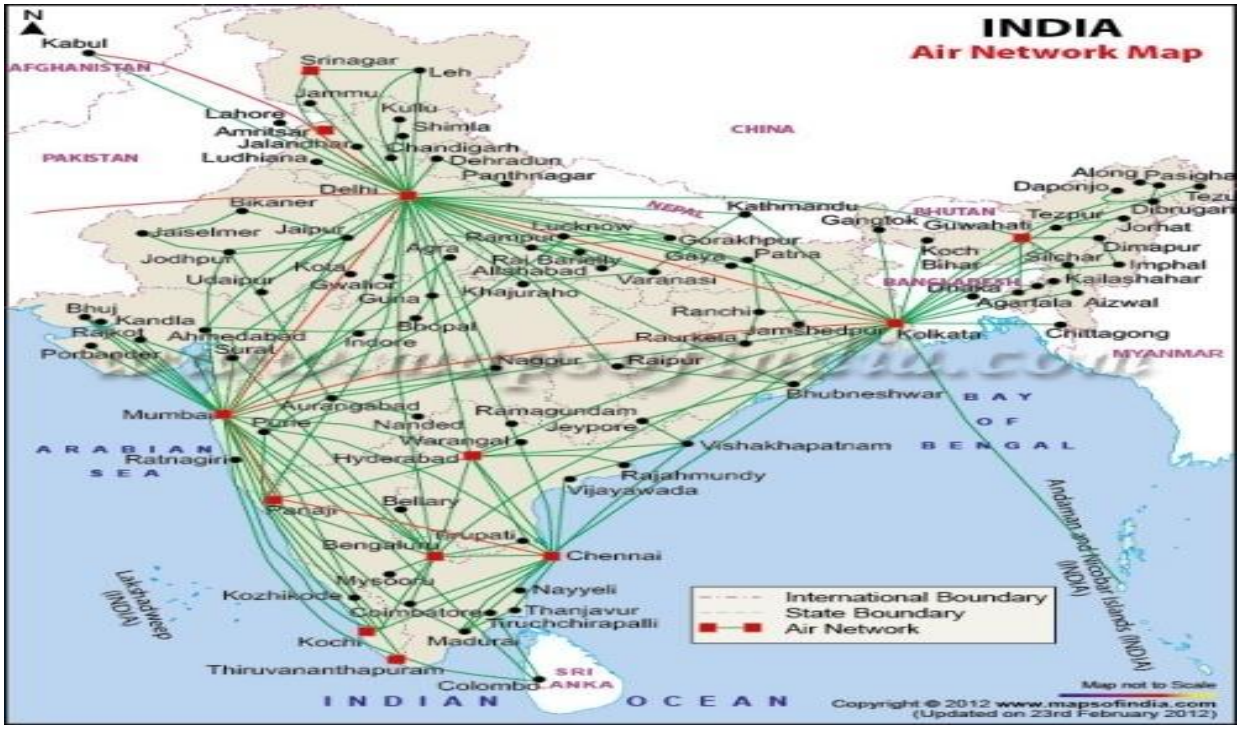
First of all I shall study all the papers and books published so for containing the study of Barnett's conjecture as well as cubic planar three connected bipartite Hamiltonian and hypo Hamiltonian graphs.

For the proof of different results, we shall use crossing number technique and for the proof of Barnett's conjecture we use different techniques which has been describes

already by different mathematicians how to settle Barnett's conjecture like Barnett's conjecture holds if and only if for any arbitrary edge in a Barnett's graph is a part of some Hamiltonian cycle. It is difficult to say whether any of the technique described so for will aid in settling Barnett conjecture. Certainly many of them seems to be useful and worth extending. One strategy is to keep chipping away at it; if Barnett conjecture is true then Godey's result can be extended to show that successively large and large subsets of Barnett graphs are Hamiltonian

7. Importance of research proposal

In the present thesis we are dealing with Hamiltonian graphs, because Barnett's conjecture is related to Hamiltonian graphs which state that every cubic planar bipartite three connected graph is Hamiltonian. There are several applications of such graphs in our day to day life. Present thesis is important because it gives us some methods to solve some of day to day life problems in an efficient way. Barnett's conjecture is one of the oldest unsolved problems in graph theory since 1969 in this thesis we try to solve this problem by using some cleaver techniques. There are other day to day problems related with these graphs like, Road connectivity, Air ways, internet connectivity, traveling salesman problem, Chinese postman problem chip designing and what not everything in our world is linked: cities are linked by street, rail and flight networks. Pages on the internet are linked by hyperlinks. The different components of an electric circuit or computer chip are connected and the paths of disease outbreaks form a network. Scientists, engineers and many others want to analyses, understand and optimize these networks. And this can be done using graph theory.



For example, mathematicians can apply graph theory to road networks, trying to find a way to reduce traffic congestion. An idea which, if successful, could save millions every year which are lost due to time spent on the road, as well as mitigating the enormous environmental impact. It could also make life safer by allowing emergency services to travel faster and avoid car accidents in the first place. These Intelligent Transportation Systems could work by collecting location data from smart phones of motorists and telling them where and how fast to drive in order to reduce overall congestion.

Graph theory is already utilized in flight networks. Airlines want to connect countless cities in the most efficient way, moving the most passengers with the fewest possible trips: a problem very similar to the Travelling Salesman. At the same time, air traffic controllers need to make sure hundreds of planes are at the right place at the right time and don't crash: an enormous task that would be almost impossible without computers and graph theory. One area where speed and the best connections are of crucial importance is the design of computer chips. Integrated circuits (ICs) consist of millions of transistors which need to be connected. Although the distances are only a few millimeters, it is important to optimize these countless connections to improve the performance of the chip.

In recent years, there has been another important use of Graph Theory: the internet. Every page in the internet could be a vertex in a graph, and whenever there is a link between two pages, there is an edge between the corresponding vertices. The resulting graph is of course *very, very* large. Early web search engines had a very big problem: they could search the web for a particular keyword, but they couldn't determine whether a page is "good" or just spam. If you searched for 'London', you might get hundreds of websites of small shops in London, or people who live there, before the official london.gov website. Google found a solution to this: any page that is very good will has many other pages linking to it. Pages that are rarely visited, or not very interesting, will be very "lonely" in the internet graph with only few other pages linking to it. This gives a way to *rank* websites and allows Google to display the best results at the beginning

There is another digital graph, of which you yourself are a part: Face book. All the users form vertices and whenever two users are friends they are linked by an edge. Graph theory can help web developers improve the performance of social networking sites, and it can help us understand Face book better.

8. Literature review

A planar graph is an undirected graph that can be embedded into the Euclidean plane without any crossings. A planar graph is called polyhedral if and only if it is three vertex connected, that is, if there do not exists two vertices the removal of which would disconnect the rest of the graph. A graph is bipartite if its vertices can be colored with two different colors such that each edge has one end point of each color. A graph is Cubic if each vertex is the end point of exactly three edges. And a graph is Hamiltonian [6] [7] if there exists a cycle that pass exactly once through each of its vertices. Self loops and parallel edges are not allowed in these graphs. Barnett's conjecture states that every cubic polyhedral graph is Hamiltonian. P.G. Tait in (1884) Conjectured that every cubic polyhedral graph is Hamiltonian; this came to be known as Tait's conjecture. It was disproved by W.T. Tutte (1946), who constructs a counter example with 46 vertices; other researchers later found even smaller counterexamples, however, none of these counterexamples is bipartite. Tutte himself conjectured that every cubic 3-connected bipartite graph is Hamiltonian [12] to [19] but this was shown to be false by discovery of a counterexample, the Horton graph [1] .David W. Barnett (1969) proposed a weakened combination of Tait's and Tutte's conjecture, stating that every cubic bipartite polyhedral graph is Hamiltonian this conjecture first announced in [4] and later in [30]. In [3], Tutte proved that all planar 4-connected graphs are Hamiltonian ,and in [2] thomassen extended this result by showing that every planer 4-connected graph is Hamiltonian connected, that is for any pair of vertices, there is a Hamiltonian path with those vertices as endpoints for complete details regarding cubic planar three connected bipartite graphs see [7] to [11].In [20] Holton confirmed through a combination of clever analysis and computer search that all Barnett graphs with up to and including 64 vertices are Hamiltonian. in an announcement [20,2], McKay Used computer search to extend this result to 84 vertices this implies that if Barnett conjecture is indeed false than a minimal counterexample must contain at least 86 vertices, and is therefore considerable larger than the minimal counterexample to Tait and Tuttle conjecture. This is not all we know about a possible counterexample; another interesting result is that of Fowler, who in an unpublished manuscript [26] provided a list of sub graphs that cannot appear in any minimal counterexample to Barnett's conjecture.

Goody in [29] considered proper subsets of the Barnett graphs and proved the following.

Theorem 1: Every Barnett graph which has faces consisting exclusively of quadrilaterals, and hexagons is Hamiltonian, and further more in all such graphs, any edge that is common to both a quadrilateral and a hexagon is a part of some Hamiltonian cycle.

Theorem 2 : Every Barnett graph which has faces consisting of 7 quadrilaterals, I octagon and any number of hexagons is Hamiltonian, and any edge that is common to both a quadrilateral and an octagon is a part of some Hamiltonian cycle.

In [6] Jensen and Toft reported that Barnett conjecture is equivalent to following,

Theorem 3: Barnett conjecture is true if and only if for every Barnett graph G, it is possible to partition its vertices in to two subsets so that each induced an acyclic sub graph of G. (this theorem is not correct)

Theorem 4 [25]: The edges of any bipartite graph G can be colored with Δ colors, where Δ is the minimum degree of any vertex in G.

Theorem 5[22]: Barnett conjecture holds if and only if any arbitrary edge in a Barnett graph is a part of some Hamiltonian cycle.

Theorem 6 [5]: Barnett conjecture holds if and only if for any arbitrary face in a Barnett graph there is a Hamiltonian cycle which passes through any two arbitrary edges on that face.

Theorem 7 [38]: Barnett conjecture holds if and only if for any arbitrary face in a Barnett graph and for any arbitrary edges e_1 and e_2 on that face there is a Hamiltonian cycle which passes through e_1 and avoids e_2

If Barnett conjecture is true then Godey's result can be extended to show that successively large and large subsets of Barnett graphs are Hamiltonian.

9. Research gapes identified in the proposed field of investigation.

Barnett's conjecture has not been proved yet because it is difficult to say whether any of the technique described so far will aid in settling Barnett conjecture. Certainly many of them seems to be useful and worth extending.

10. Chapter wise detail

Chapter 1 Introduction.

In chapter one, we give introduction of the thesis, structure of the thesis goals

And result of the thesis as well as we give basic definitions of the graph theory

Used in the thesis.

Chapter 2 Cubic planar three connected bipartite Hamiltonian graphs.

In chapter two, we give definitions and properties of cubic planar bipartite

Three connected Hamiltonian graphs along with some new definitions and

Develop some techniques to prove some new results.

Chapter 3 Barnett's conjecture on Hamiltonian graphs.

In this chapter we give introduction and literature reviews of Barnett's conjecture.

We will state some new lemmas with proofs related to Barnett's conjecture and

Finally we will try to give the proof of Barnett's conjecture.

Chapter 4 Hypo Hamiltonian graphs, planar and non planar graphs.

In this chapter we will extend Euler result of number of regions in any planar graphs To all simple non planar graphs and prove some new results on hypo Hamiltonian Graphs.

11. Expected duration of the investigation

From July 2014- Jan 2015, literature reviews and publications of some research Work related to the thesis along with course work.

From Jan 2015- July 2015, try to complete some papers for possible publication.

From July 2015- Jan 2016 start to write the thesis and published other pending Work and finally I shall submit my thesis in January 2016

12. Facilities available for the investigation at scholar end and facilities required on campus.

First of all I shell obey all the rules and regulations of the university and “I Will Let No Stone Unturned in doing the research in my field” for this I require internet facility, library facility , T.A. and D.A for attending seminars within and off campus.

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